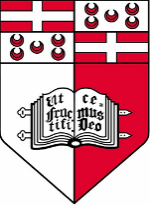
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**SOR2220**

**Statistical Inference I**

Assignment

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Given pdf

for where

1. Work out the cdf , three middle quartiles, mean, variance, skewness and kurtosis. Give full derivations.

*Assuming log is the natural logarithm*

CDF: F(x) =

Lower Quartile: Let 0.25 = F (π0.25)

Median: Similarly, solving for F (π0.5) = 0.5 gives us that

Upper Quartile: Solving for F (π0.75) = 0.75 gives us that

Mean:

Second Moment:

Third Moment:

Fourth Moment:

Variance:

Skewness:

Kurtosis:

2. Compare the corresponding values for mean, variance, skewness and fourth moment using these estimated parameters and compare with actual sample values.

Given Data

First data set

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 2.1657 | 1.9058 | 1.8769 | 1.3573 | 1.9507 | 1.0395 | 1.3739 | 1.6425 | 2.1955 | 2.2984 |
| 1.8633 | 1.6742 | 1.7362 | 2.0143 | 2.0982 | 1.9918 | 2.2763 | 1.6038 | 1.5679 | 1.9707 |
| 2.1808 | 1.7894 | 1.7878 | 1.4078 | 2.1398 | 1.5931 | 2.2361 | 1.4292 | 1.7559 | 1.3519 |
| 1.3391 | 2.0084 | 2.2859 | 1.3248 | 1.1917 | 1.9435 | 2.2788 | 1.3706 | 2.2873 | 1.8899 |
| 1.4080 | 1.7373 | 1.8725 | 2.1554 | 1.9642 | 1.1309 | 1.1206 | 2.0024 | 2.2532 | 1.2082 |
| 1.0080 | 1.2077 | 2.1333 | 1.0511 | 1.7826 | 2.0257 | 1.3193 | 2.1644 | 2.0941 | 1.7887 |
| 1.1595 | 1.4214 | 1.7078 | 1.0675 | 1.7450 | 1.0928 | 1.6246 | 2.0423 | 1.6252 | 2.0601 |
| 1.8356 | 2.2157 | 1.0378 | 1.6429 | 1.4829 | 1.8909 | 1.7650 | 1.0666 | 2.0945 | 1.6133 |
| 1.4636 | 1.8279 | 1.3880 | 1.1184 | 1.2696 | 1.7207 | 1.6745 | 1.7858 | 1.6511 | 1.1484 |
| 2.0457 | 1.7676 | 1.7807 | 2.2769 | 1.4908 | 1.7565 | 1.8714 | 1.8265 | 1.5697 | 1.8252 |
| 1.8909 | 2.2297 | 1.1774 | 1.8749 | 1.9246 | 2.0028 | 1.7229 | 1.3913 | 1.9668 | 1.8872 |
| 1.2964 | 1.4043 | 1.6881 | 1.9073 | 1.7490 | 1.4302 | 2.0645 | 1.5929 | 1.3812 | 1.2145 |
| 1.4421 | 2.2011 | 1.2891 | 1.5016 | 1.8499 | 1.3799 | 1.9705 | 2.0817 | 1.2909 | 2.1483 |
| 1.2310 | 1.7892 | 1.4983 | 1.6896 | 2.0283 | 2.0279 | 1.8040 | 1.4670 | 1.3801 | 1.7048 |
| 1.7969 | 2.0033 | 1.1914 | 1.0309 | 1.2733 | 2.2553 | 1.2632 | 1.7070 | 1.4425 | 1.4322 |
| 1.2438 | 1.9401 | 1.6307 | 1.3261 | 1.7129 | 1.2768 | 1.8710 | 2.0525 | 1.0752 | 1.4799 |
| 2.1379 | 1.2369 | 1.9805 | 2.1406 | 2.0334 | 1.5676 | 2.2168 | 1.6579 | 1.7823 | 1.9193 |
| 1.0569 | 2.2703 | 2.0367 | 1.1216 | 1.4656 | 1.1697 | 2.2812 | 2.1119 | 2.2356 | 1.7199 |
| 2.1314 | 1.6260 | 2.2183 | 1.4789 | 2.1183 | 2.2266 | 1.9854 | 1.9312 | 2.0600 | 1.6506 |
| 1.3726 | 1.1361 | 2.0244 | 2.1044 | 1.9348 | 2.0773 | 1.3081 | 1.1532 | 1.3847 | 1.4986 |
| 2.0221 | 1.1875 | 1.5077 | 1.8547 | 1.5024 | 1.9935 | 2.1425 | 2.2784 | 1.9537 | 1.7998 |
| 1.1026 | 2.2689 | 2.1657 | 2.2041 | 1.7664 | 2.0679 | 1.2206 | 1.8004 | 1.0291 | 1.0290 |
| 1.9673 | 1.8051 | 1.7511 | 2.1697 | 2.1171 | 1.8090 | 2.0089 | 1.3371 | 2.0378 | 1.7840 |
| 1.3031 | 1.6999 | 1.4204 | 1.3901 | 1.3868 | 1.5458 | 1.0879 | 1.1678 | 1.5069 | 2.2280 |
| 2.2940 | 1.8939 | 1.2797 | 2.2532 | 1.0369 | 1.2695 | 1.8995 | 1.1131 | 1.4943 | 1.6974 |
| 1.7402 | 1.3521 | 1.0829 | 1.6608 | 1.8495 | 2.1722 | 1.1080 | 1.0485 | 2.0342 | 1.5024 |
| 1.7663 | 1.9525 | 1.2835 | 2.1747 | 1.7671 | 2.1899 | 2.2100 | 1.5098 | 1.5747 | 2.0775 |
| 1.4261 | 1.3907 | 2.2167 | 1.6764 | 1.3636 | 2.0263 | 1.2849 | 1.7888 | 1.3527 | 1.2426 |
| 1.5671 | 1.8805 | 1.1886 | 1.8229 | 2.2102 | 1.5459 | 1.1555 | 2.1264 | 1.7967 | 1.1401 |
| 1.7346 | 2.2554 | 2.1129 | 1.0224 | 2.2282 | 1.7151 | 1.4706 | 1.8724 | 1.3555 | 1.6522 |
| 1.7386 | 1.9891 | 1.2553 | 1.9915 | 1.7058 | 1.9100 | 1.9252 | 1.8404 | 1.2715 | 1.6772 |
| 1.6815 | 2.2660 | 2.1377 | 1.6144 | 1.9556 | 1.4497 | 1.3325 | 1.3413 | 1.5945 | 2.0010 |
| 1.2588 | 2.0304 | 1.8389 | 1.2044 | 1.7401 | 1.2420 | 2.2718 | 1.4020 | 1.0386 | 1.9573 |
| 1.3477 | 1.7504 | 1.3229 | 1.8361 | 1.4447 | 1.4754 | 1.0733 | 1.9022 | 1.6244 | 1.7601 |
| 1.8941 | 1.6539 | 2.0711 | 1.0091 | 1.0181 | 1.5272 | 1.7097 | 1.5030 | 1.0063 | 1.3352 |
| 1.8479 | 1.8861 | 2.0595 | 2.2321 | 1.4551 | 1.2367 | 1.8891 | 1.7481 | 1.0826 | 1.2630 |
| 1.3005 | 1.4741 | 1.1678 | 1.9484 | 1.9192 | 1.3493 | 1.6013 | 1.2138 | 1.3104 | 1.5978 |
| 2.1511 | 1.6497 | 1.2845 | 1.4265 | 1.2602 | 1.7876 | 1.1038 | 1.6613 | 1.3280 | 1.6424 |
| 1.7536 | 1.9178 | 1.8439 | 2.0207 | 1.3625 | 1.0936 | 1.8766 | 2.2917 | 1.8410 | 2.2721 |
| 2.2715 | 2.0994 | 1.4979 | 1.9096 | 2.1623 | 1.6170 | 1.3469 | 1.5940 | 1.3711 | 2.2178 |
| 1.4450 | 1.4899 | 1.4203 | 1.4426 | 1.8474 | 1.9719 | 1.8759 | 1.1120 | 1.8006 | 1.4753 |
| 2.2430 | 1.9081 | 2.0617 | 1.7632 | 1.8961 | 2.1642 | 1.5880 | 1.4551 | 1.7815 | 2.1453 |
| 1.8496 | 1.7264 | 1.8604 | 1.1092 | 1.1243 | 1.8424 | 2.0695 | 1.7834 | 1.5149 | 1.7929 |
| 2.0469 | 1.2637 | 1.9426 | 1.9320 | 1.3526 | 1.6766 | 2.1639 | 2.0050 | 2.2887 | 1.7736 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1.1147 | 1.4713 | 1.9554 | 2.2135 | 2.0510 | 2.0916 | 2.0511 | 1.4891 | 1.2501 | 2.1222 |
| 1.8300 | 1.1830 | 2.2238 | 1.6786 | 1.8751 | 1.4940 | 1.9613 | 1.7122 | 2.1325 | 2.0601 |
| 2.1874 | 2.1544 | 1.2068 | 1.5550 | 1.1267 | 1.5647 | 1.6211 | 1.8746 | 2.0592 | 1.7641 |
| 1.7318 | 1.9338 | 1.8488 | 2.2073 | 1.9083 | 1.2343 | 1.8926 | 2.0201 | 1.7495 | 1.0035 |
| 1.7598 | 2.1046 | 1.7073 | 1.0222 | 1.7438 | 1.9041 | 1.1395 | 1.9513 | 1.0357 | 1.5567 |
| 2.0723 | 2.2396 | 1.7560 | 2.2206 | 2.2412 | 1.0480 | 1.5703 | 1.2326 | 1.4151 | 1.6406 |

Second Data set

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 2.4717 | 2.4137 | 2.9092 | 1.0758 | 1.4668 | 2.1044 | 1.8557 | 1.2905 | 2.9156 | 2.9671 |
| 1.3732 | 1.4920 | 3.2963 | 2.3823 | 3.1793 | 2.3836 | 3.2381 | 1.6258 | 1.7619 | 3.0744 |
| 2.3207 | 1.3655 | 1.9133 | 2.6908 | 1.9483 | 1.2235 | 1.3682 | 1.3944 | 2.3950 | 2.9303 |
| 1.9596 | 2.7718 | 1.4755 | 2.5161 | 1.3550 | 2.1690 | 2.7884 | 2.8747 | 2.1059 | 2.7378 |
| 1.1441 | 2.5152 | 3.2825 | 1.1646 | 2.2673 | 2.7931 | 1.8653 | 2.4637 | 2.5274 | 1.6350 |
| 1.3474 | 1.8233 | 2.5759 | 1.3882 | 1.0828 | 1.3422 | 1.6862 | 2.5300 | 1.6393 | 3.1789 |
| 1.9314 | 1.1509 | 3.1509 | 2.7454 | 2.2831 | 2.6212 | 2.2719 | 2.5406 | 1.6929 | 2.9515 |
| 2.0810 | 1.9981 | 2.4788 | 2.1661 | 1.9450 | 2.9145 | 1.9688 | 2.2805 | 2.6008 | 2.9408 |
| 1.8000 | 2.0883 | 2.5062 | 1.9177 | 2.7573 | 2.2046 | 3.1176 | 1.6961 | 3.2391 | 2.7937 |
| 2.9746 | 2.2543 | 1.9138 | 1.7015 | 1.4113 | 1.0134 | 1.8562 | 1.2532 | 1.6884 | 2.1170 |

|  |  |  |
| --- | --- | --- |
|  | First Data Set | Second Data Set |
| Sample Size: | 500 | 100 |
| Lower Quartile | 1.39 | 1.69 |
| Median | 1.745 | 2.17 |
| Upper Quartile | 1.99 | 2.7025 |
| Maximum | 2.3 | 3.3 |
| Minimum | 1 | 1.01 |
| Range | 1.29 | 2.28 |
| Mean | 1.6954 | 2.1685 |
| Mean of X2 | 3.005 | 5.0941 |
| Mean of X3 | 5.5274 | 12.7395 |
| Mean of X4 | 10.4845 | 33.4174 |
| Variance | 0.131 | 0.395 |
| Skewness | -0.183 | -0.11 |
| Kurtosis | -1.073 | -1.068 |

1. **Method of moments**

3. Construct three estimators for *a*, one of them using some order statistic, and work out their values for the two datasets.

Discuss the statistical properties of each estimator.

Do the two samples come from the same population?

Let

Solving for the first dataset where using software we get that a -> 2.29113.

Similarly for the second dataset where we get that a -> 3.29591.

**Properties of the Estimator**

1. Unbiasedness

Since all sample values come from the same distribution, we know that , thus ; which shows that the mean estimator used is unbiased.

1. Variance